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D-branes in 2D lorentzian black hole

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Abstract: We study D-branes in the lorentzian signature 2D black hole in string theory. We use the technique of gauged WZW models to construct the associated boundary conformal field theories. The main focus of this work is to discuss the (semi-classical) world-volume geometries of the D-branes. We discuss comparison of our work with results in related gauged WZW models.

Keywords: D-branes, Conformal Field Models in String Theory, Black Holes in String Theory.

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1. Introduction

In view of the recent activity in two-dimensional string theory, much of it revolving around the new interpretation of the $c = 1$ matrix model as (a scaled limit) of the open string theory on unstable D0-branes in Liouville theory, it is natural to study the possible branes in other 2-dimensional string backgrounds. The case of the euclidean black hole has been discussed in a recent paper [1]. In this work, we investigate the lorentzian black hole string theory using primarily semi-classical methods based on the $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ conformal field theory. A more precise study will entail the construction of the corresponding boundary states [2].

Another motivation in starting this study was that our preliminary investigation revealed that the D-branes in the lorentzian geometry are time-dependent. Since time dependent solutions to string theory are somewhat at a premium, such D-branes are likely to be interesting.

In view of conjectures that the lorentzian black hole geometry is related to the phase space picture of the $c = 1$ matrix model [3, 4], such branes could help in studying these questions.

D-branes have proved to be extremely useful in string theory in general in studying the geometry of backgrounds, especially near singularities. It is but natural to attempt to study black-hole singularities using D-branes. Our work could be regarded as a first step towards such an end.

The outline of this paper is as follows. In the first section, we provide a short summary of the construction of this string theory background as the coset $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{U}(1)$. This coset construction allows us to use the the geometrical methods of [5]–[9] to analyse the allowed boundary conditions consistent with conformal invariance. This general procedure is described (albeit rather briefly) in the second section.

The third section describes the geometry of the various branes found by the preceding technique in this string background. It is to be noted that the semi-classical geometrical analysis presented in this section is valid only for large level of the CFT. The exact string background is obtained by setting the level $k = 9/4$, when loop corrections are substantial [10, 11]. The idea then, is to use the semiclassical understanding to construct the boundary states along the lines of [12, 13, 1]. We describe the various allowed boundary conditions, and describe the world-volume geometries of the of the branes. We find three kinds of D-branes namely, D(-1), D0- and D1-branes which are both emitted from the white hole and fall into the black hole.

In the next sections, we compare these D-branes with those found in the euclidean black hole geometry and that of the Parafermion theory. In the latter case, one is really comparing boundary CFT's (not string backgrounds), but we deem this a worthwhile exercise since this could be useful in the boundary state construction.

Recalling that the extended (lorentzian) black hole geometry possesses a duality [10] which exchanges the region in front of the horizon with the region behind the singularity, we then briefly examine the duality relations between these branes.

Lastly we include a summary and list some natural questions and directions for further study.

The appendix contains some useful co-ordinate charts for $\mathrm{SL}(2,\mathbb{R})$ and a brief discussion of the BCFT lagrangian for gauged WZW models.

2. D-branes in a gauged WZW model

D-branes in gauged G/H WZW models have been studied in a series of papers including [13, 14, 15, 5, 6, 16, 9]. The conclusions may be summed up by the statement: the allowed Dirichlet boundary conditions for open strings consist of products of (twined) conjugacy classes of G with those of H projected down to the coset (keeping track of the product factors and after possible translations by elements of H).

This result is explained [5, 6, 9, 8] as follows. For the present purpose, we shall assume that H is the subgroup $H \times H^{-1}$ or $H \times H$ of the $G \times G$ symmetry of the WZW model

— the former is the vectorial gauging and the latter the axial gauging of H. We will also assume that H is abelian (the case relevant for the black-hole is $H = \mathrm{U}(1)$), in which case the conjugacy classes of H are the various points of H.

The boundary conditions consistent with the symmetries of the ungauged WZW model are those for which the worldsheet boundaries are restricted to the (twined) conjugacy classes of the group manifold G [12]. In this case, the regular conjugacy classes and the twined conjugacy classes correspond to A-type and B-type branes.

When this sigma model is gauged the allowed boundary conditions must be consistent with the symmetry being gauged. This can happen in the following way. For specificity, we shall consider the axial gauging. In this case, consider the twined conjugacy class $C_g^\omega = \{\omega(h)\, g\, h^{-1}, \forall h \in G\}$ where ω is an outer automorphism that acts on H as $h \in H \to \omega(h) \equiv \omega\, h\, \omega^{-1} = h^{-1}$ i.e. ω takes h to its inverse. Under the axial symmetry

$$C_g^\omega \to k\, C_g^\omega\, k = \{\omega(k^{-1} h)\, g\, (h^{-1}\, k)\, |\forall h \in G\}$$

and hence this set of boundary conditions is left invariant. We could have also translated the C_g^ω by elements $k_0 \in H$ as $k_0 C_g^\omega$ (since H is assumed to be abelian; in the general case one considers products of conjugacy classes of G and H).

The gauging operation results in a target space which is a coset under the equivalence relation $g \sim h\, g\, h$, and hence the set of boundary conditions becomes the projection of $k_0\, C_g^\omega$ to the coset. Note that there is no loss of generality in restricting to left translations (right translations are equivalent to left translations).

This set of boundary conditions does not preserve the $H \times H^{-1}$ current algebra symmetry (target space isometry) of the gauged sigma model (the twisted conjugacy class is not invariant under $C_g^\omega \to k\, C_g^\omega\, k^{-1}$). Since this symmetry is spontaneously broken there are zero modes corresponding to translations along the isometry direction (this corresponds to left translations by elements $k_0 \in H$). We will call these branes A-type (analogous to the A-branes of [13] because the A-branes of that paper are of A-type w.r.t the vectorial gauging).

On the other hand, we could also consider the regular conjugacy classes C_g of G. Under the action of the axial symmetry, the conjugacy classes are left translated by elements of H : $l_0\, C_g \to k\, l_0\, C_g\, k = k l_0 k C_g = C_{l_0}^{\omega,H} C_g$ (for generality we have included a translation by an element $l_0 \in H$; the translated set is then a product of a twined conjugacy class of H and the regular conjugacy class of G) Thus, if we consider the set

$$\tilde{C}_g = \cup_k \{k\, l_0\, C_g\, k \,|\, k \in H\}$$

this set of boundary conditions is invariant under the axial gauging. As before, we can translate by elements $k_0 \in H$ (and again left and right translations are equivalent). Observe that the union is over a fixed G-conjugacy class (which is assumed to be connected). This set is then projected down to the coset by the gauging. In this projection, the final brane world-volume that emerges is the restriction of the regular conjugacy class C_g to the coset. These B-type brane-worldvolumes are invariant under the isometry.

We can also understand the presence of the two types of branes by looking at quantum states in the parent theory. The A-type branes are the D-brane states of the parent theory

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which are invariant under the symmetry being gauged. Geometrically this simply means that the A-brane world-volumes are preserved under the symmetry being gauged. The other set of branes of the coset theory are obtained simply by superposing branes of the parent theory (that are *not* invariant under the gauge symmetry) to construct states which are invariant under the gauge symmetry.

On the other hand, if we consider the vectorial gauging of H, then the roles of the A-type and the B-type branes are reversed. The regular conjugacy classes are left invariant by the gauge symmetry, while the twined conjugacy classes are translated. Thus by similar arguments, we can construct branes in the coset theory, as projections of (twined) conjugacy classes after suitable translations (and superpositions).

When the symmetry that is being gauged is the axial symmetry, the two types of branes preserve different amounts of the target space isometry of the coset (which is the vectorial symmetry but possibly anomalous). The A-type branes arise from the twined conjugacy classes — which break the vectorial symmetry in the parent theory itself. Hence in the coset, they are not invariant under the isometry. But because they break this symmetry spontaneously, there is a family of such states

The B-type brane world-volumes are invariant under the vectorial symmetry in the parent theory, and hence in the coset theory, their world-volume is invariant under this symmetry.

We can write down the BCFT sigma model lagrangian explicitly and show that these boundary conditions are consistent with conformal invariance (for a brief discussion, see the appendix). Note however that these sets of boundary conditions preserve one half of the current algebra of the bulk gauged sigma model. This is perhaps sufficient to establish the claim of conformal invariance.

3. The lorentzian black hole

The black hole solution to string theory in two dimensions was first constructed as a solution to the beta function equations of 2-D string theory by [17]. Subsequently, Witten [18] constructed a CFT description of this background in the form of a gauged WZW model. It is this latter construction that we use below.

The lorentzian black hole is obtained by gauging a non-compact axial U(1) symmetry of the $\mathrm{SL}(2,\mathbb{R})$ WZW model. We shall briefly outline the procedure — for details refer to [18, 10].

The symmetry that is being gauged corresponds to a hyperbolic subgroup of $\mathrm{SL}(2,\mathbb{R})$ which acts on $g = \left(\begin{smallmatrix} a & u \\ -v & b \end{smallmatrix}\right) \in \mathrm{SL}(2,\mathbb{R})$ as $\delta g = \epsilon(\sigma_3\, g + g\, \sigma_3)$, i.e.

$$\begin{aligned} \delta a &= 2\epsilon a \qquad & \delta u &= 0 \\ \delta b &= -2\epsilon b \qquad & \delta v &= 0\,. \end{aligned} \tag{3.1}$$

In gauging the $\mathrm{SL}(2,\mathbb{R})$ theory, one has to choose a gauge fixing condition. As Witten has argued, this is a subtle issue since there is no single gauge choice which gives rise to a globally two dimensional target space.

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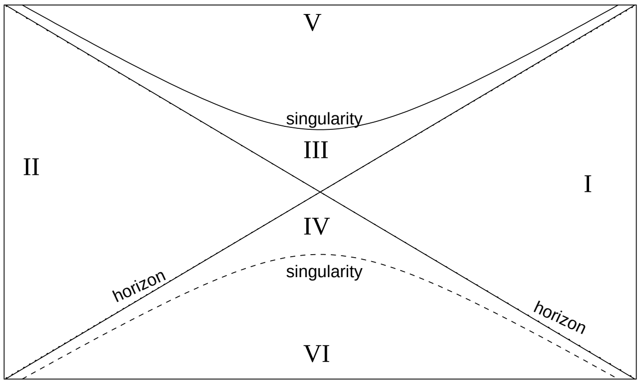

Figure 1: The lorentzian black hole geometry.

In the region $(1 - uv) > 0$, $ab > 0$ and hence a natural gauge fixing condition is $a = b$. Upon integrating out the gauge fields (which appear quadratically), we obtain the sigma model action

$$L = -\frac{k}{4\pi} \int d^2x \sqrt{h}\, \frac{h^{ij}\, \partial_i u\, \partial_j v}{(1 - u\, v)}\,. \tag{3.2}$$

In the region $(1 - uv) < 0$ however, a good gauge fixing condition is $a = -b$ (because $ab < 0$). When $uv = 1$ either $a = 0$ or $b = 0$ or both, hence we cannot gauge transform a field configuration to the gauge slice (for either gauge choice). Although the gauge fixing condition is singular, the sigma model is itself non-singular.

The target space geometry of the sigma model so obtained is as shown in the figure 1

In the figure, the diagonal lines $uv = 0$ form the horizon, while $uv = 1$ is the singularity (the Ricci scalar diverges as $R \sim (1-uv)^{-2}$). Regions I and II are asymptotically flat regions and in regions V and VI time flows "sideways". Constant time slices (in the asymptotically flat region) are straight lines passing through the origin with time increasing from top to bottom. Thus the black hole singularity is in the fourth quadrant (in the figure the diagonal lines are the co-ordinate axes!).

Requiring conformal invariance generates a dilaton at one loop

$$\Phi = \Phi_0 - \frac{1}{2} \ln(1 - uv) \tag{3.3}$$

where the parameter Φ_0 is related to the ADM mass of the black hole.

We can also proceed slightly differently by gauging the vectorial action of H [10], i.e. which acts on $g \in \mathrm{SL}(2,\mathbb{R})$ as $\delta g = \epsilon(\sigma_3\, g - g\, \sigma_3)$, i.e.

$$\begin{aligned} \delta u &= -2\epsilon u \qquad & \delta a &= 0 \\ \delta v &= 2\epsilon v \qquad & \delta b &= 0\,. \end{aligned} \tag{3.4}$$

Thus a natural gauge choice in this region is $u = v$, and as before we have a gauge singularity at $ab = 1$. In this case, the black hole geometry is covered by the (a, b) coordinates of $\mathrm{SL}(2,\mathbb{R})$ (the sigma model so obtained has the same target space metric as before, with (a, b) replacing (u, v) and $ab = 1$ being the singularity).

Thus, we have two descriptions of the black hole geometry: one obtained by gauging the axial U(1) and another obtained by gauging the vectorial U(1). These two descriptions are dual to each other [19, 10]. We will make use of both descriptions.

While this analysis is performed at the leading order in α', the exact background to all orders is known [10, 11, 20]. However we shall restrict our investigation to the leading order. The geometrical description presented in the subsequent sections makes sense at large k. The exact string background is obtained when $k = 9/4$ when α' corrections are substantial.

In the following, it will be convenient to use another set of co-ordinates to cover the black hole — which are natural from the SL$(2,\mathbb{R})$ point of view as described in the appendix. In the regions I and II, the co-ordinate transformation is $u = -\sinh\rho\, e^{-t}$, $v = \sinh\rho\, e^{t}$ and the metric is

$$ds^2 = k(d\rho^2 - \tanh^2\rho\, dt^2)\,. \tag{3.5}$$

Analytically continuing $t \to i\tau$, we get the euclidean black hole which also has a description as a gauged WZW model.

The regions $0 < uv < 1$ of the black hole can be analytically continued to the Parafermion (Pf) theory SU(2)/ U(1). In this region (III & IV in the figure), the co-ordinate change is $u = \sin\rho\, e^{-t}$, $v = \sin\rho\, e^{t}$ and the metric can be written as

$$ds^2 = -k(d\rho^2 - \tan^2\rho dt^2)\,. \tag{3.6}$$

The analytic continuation is performed by $t \to i\tau$ and $k \to -k$ which gives us the target space of the parafermion theory at level k. This target space is topologically the unit disk, and has a curvature singularity at the boundary of the disk. In this continuation, the black hole singularity which is a curvature singularity maps to the boundary of the disk and the horizon of the black hole maps to the center of the disk.

The "natural" co-ordinates for regions V &VI behind the singularity are $u{=}\pm\cosh\rho e^{-t}$, $v = \pm\cosh\rho e^{t}$.

4. D-branes in the lorentzian black hole

Using the procedure outlined in section 2, and the *axially* gauged WZW construction of the black hole CFT, we can identify the various D-branes obtainable as BCFT's in this geometry.

The regular conjugacy classes of SL$(2,\mathbb{R})$ are well known and their topologies have been described in e.g. [21]. SL$(2,\mathbb{R})$ has only one outer automorphism upto conjugation [22], which we take to be conjugation by σ_1, and hence a one parameter family of twined conjugacy classes

$$C_g^{\sigma,G} = \{\sigma_1 h \sigma_1 g h^{-1}, h \in G\}\,.$$

These are characterised by a class invariant $\mathrm{Tr}(\sigma_1 g_0) = 2\kappa$ and form a connected submanifold in SL$(2,\mathbb{R})$ for each κ. The other outer automorphisms which are SL$(2,\mathbb{R})$ conjugates of σ_1 give rise to twined c.c. that are translates of the ones above. Hence, it is sufficient to restrict our attention to these and their translates.

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Since the twined conjugacy classes are preserved under the gauge symmetry, their geometry is determined simply by projection to the coset. These can also be (left) translated by elements in H. Thus they are characterised by two parameters κ, the class invariant, and the translation $l_0 \in H$ and give rise to A-branes in these theories.

The B-type branes are obtained by projecting the set $C^{\omega,H}_{l_0} C^G_g$ where C^G_g is a regular c.c. of $G = \mathrm{SL}(2,\mathbb{R})$. These are characterized by one real parameter, the trace of g (translations by l_0 leave the brane invariant).

We will also have occasion to use the *vectorially* gauged WZW model description. In this case the regular conjugacy classes C^G_g simply project down to the coset, being invariant under the gauge symmetry. These can also be (left) translated by $l_0 \in H$. Thus we get branes characterised by the trace $\mathrm{Tr}(g)$ and by l_0. These are the A-type branes of this theory.

The B-type branes are obtained by projections of products of H and G twined conjugacy classes $C^{\omega,H}_{l_0} C^{\omega,G}_g$, and are characterized by one parameter (the trace $\mathrm{Tr}(\sigma_1 g)$; the l_0 translations leave these branes invariant).

As described in the previous section, the region in front of the horizon can be analytically continued to the euclidean black hole. The D-branes in this latter geometry were described by [1], using an $\mathrm{SL}(2,\mathbb{C})/\,\mathrm{SU}(2)$ coset description.

Similarly, the region between the horizon and the singularity is analytically continued to the Pf-theory. In this case, the D-branes were investigated in [13].

Thus, we may analytically continue the D-brane world-volumes in these two regions and compare with the results obtained in these works. It is also to be noted that by analytically continuing the lorentzian branes, we need not get *real* branes in the euclidean theories [1].

4.1 D(-1) branes

These branes correspond to the identity conjugacy class (c.c.). In the coset, this conjugacy class projects to $(u, v) = (0, 0)$. As described earlier, left translation by a boost $l_0 = diag(e^{t_0}, e^{-t_0}) \in H$ does nothing to the location of the brane, and hence we have a single brane sitting on the (intersection of the future and past) horizon.

Another way to argue that all these "boosted" branes are equivalent is that in constructing the boundary state for these branes, the closed string one-point functions are obtained by evaluating the closed-string primaries at the location of the (point-like) branes. Since the boost does nothing to location of the branes, the boundary states are identical, and hence we have just one brane.

There is another family of such branes, which are seen in the vectorial gauging. In this case, the identity conjugacy class maps to the point $(1, 1)$. Upon translation by h_0, these branes are translated to $(a, b) = (e^{t_0}, e^{-t_0})$ still residing on the singularity $ab = 1$. A different way to assure ourselves of their existence is by comparison with the parafermion theory as discussed later in section 5. A similar set of pointlike branes are obtained when one considers the conjugacy class of $-I \in \mathrm{SL}(2,\mathbb{R})$.

[1] We are grateful to Ashoke Sen for pointing this out

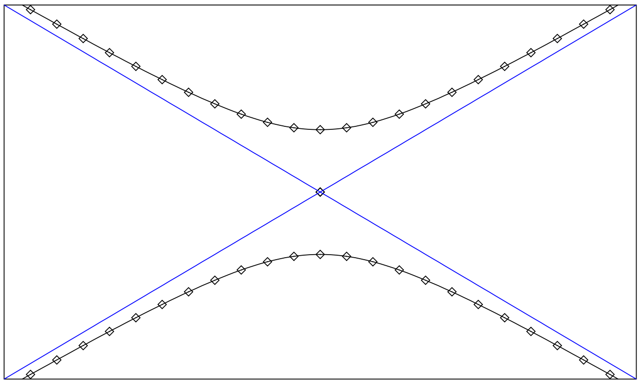

Figure 2: The point-like branes.

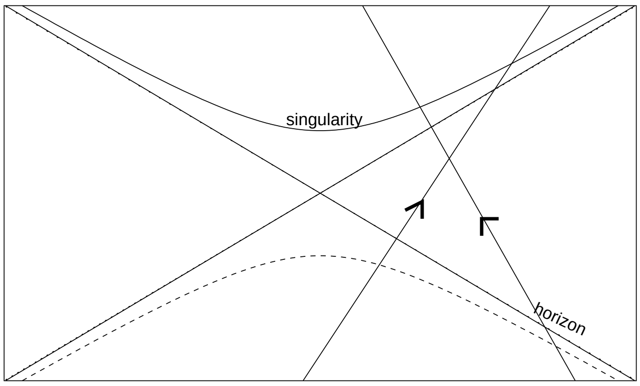

Figure 3: The D0-brane trajectories.

4.2 D0-branes

These are obtained by considering the twined conjugacy classes in the axially gauged WZW, which are characterised by the class invariant $\mathrm{Tr}(\sigma_1 g) = 2\kappa$.

Thus, projection to the coset gives a connected submanifold, whose equation in global co-ordinates is $(u - v) = 2\kappa$. Here κ can be any real number. Since the equation describing the world-volume involves only (u, v) co-ordinates, we can use global co-ordinates to discuss these branes everywhere in the (u, v)-plane, excepting at the singularities $uv = 1$ (analogous to the argument in [18], we may expect that at the singularity the CFT is well-defined, but the target space interpretation as a *D0*-brane fails).

When this is projected down to the coset, we have one relation between the two co-ords of the coset theory, thus defining a curve. For any value of κ this is a straight line in the (u, v)-plane at 45^o to the u(v)-axis. Note that it passes *through* the horizons at $uv = 0$. We can translate these by $l_0 = \mathrm{diag}(e^{t_0}, e^{-t_0}) \in H$, under which the equation $(u - v) = 2\kappa$ becomes

$$u\, e^{t_0} - v\, e^{-t_0} = 2\kappa \tag{4.1}$$

giving us a two parameter family of D0-branes.

One can argue for the existence of these branes using the Born-Infeld action also [13]. The effective metric seen by the D0-branes is

$$\frac{ds^2}{g_s^2} = du\, dv$$

which is flat. Hence, the D0-branes being point particles will move on geodesics of this metric which are straight lines. It is quite remarkable that the effective metric is that of *flat space* while the actual background has a curvature singularity even. This tempts one to speculate that the D0-branes actually pass over to the "other side" of the singularity.

This brane can also be understood in the usual r, t-co-ordinates. In terms of the (r, t)-co-ordinates which cover the region in front of the horizon, the equation defining the brane becomes

$$\sinh r \cosh t = 2\kappa \, .$$

Thus as $t \to \pm\infty$, $r \to 0$ and as $t \to 0$, $r \to r_{\max} = \sinh^{-1} \kappa$. This means that from the point of view of an asymptotic observer (whose time co-ordinate is t) this brane exited the horizon at $r = 0$ infinitely far back in the past, attained a maximum distance $r_{\max}$, and falls back into the horizon at $r = 0$ in the future. She never sees the brane actually coming *out* from the past horizon or *crossing* into the future horizon, as is usual in black hole geometries.

Note that these branes are time dependent, in the sense that the rolling tachyon is time-dependent. It will be very interesting to compare the two situations (i.e. the rolling tachyon in the Liouville theory and this) with regard to their dynamics. Another interesting aspect of these branes is that they never reach asymptotic infinity. This fact can be understood because D-branes would like to minimise their energy by staying in regions of strong coupling (the mass is $\sim e^{-\Phi(r_0)}$).

4.3 D1-branes

These space-filling branes are obtained by the projections of several of the regular conjugacy classes to the coset. Many of these worldvolumes are rather unusual in the sense that they extend behind the horizon and even behind the singularity. Another surprising feature is the presence of a boundary even though the bulk geometry is non-compact. These branes have a world-volume field strength F, turned on for stability. In 1+1-dimension, a gauge field has no propogating degrees of freedom. The F-field should probably be thought of as giving rise to a conserved charge which labels the branes.

In this case, in order to analyse stability of brane, one can study the Born-Infeld action of the brane with world-volume F_{uv} field present (the question is whether there is a solution to the BI- equations of motion of this brane with this F-field which would (perhaps) stabilise the brane [23]).

$$\begin{aligned} S_{DBI} &= \int e^{-\Phi} \sqrt{-det(g + F)} \\ &= \int e^{-\Phi} \sqrt{\frac{1}{(1-uv)^2} - F_{uv}^2} \end{aligned} \tag{4.2}$$

The equation for F gives

$$F_{uv}^2 = \frac{-det(g)\, f^2}{f^2 + e^{-2\Phi}} = \frac{1}{(1-uv)^2}\frac{f^2}{(1+f^2-uv)}\,. \tag{4.3}$$

In this equation, g and Φ are the (pullbacks) of the closed string metric and dilaton (and we use static gauge in u,v). It is easily seen that for both values of F, we do not have an imaginary BI-action i.e. the electrical F remains subcritical in the entire world-volume (in the region $(1-uv) < 0$). Further, note that F blows up at the singularity $uv = 1$ and at $uv = 1 + f^2$ and, for $uv > 1 + f^2$, F becomes imaginary. We can interpret this to mean that the D1-branes in regions I-IV terminate at the singularity, while those in the "dual" regions V,VI are bounded by $uv = 1$ and $uv = 1 + f^2$. A precise statement will however require the construction of the boundary states from which the boundaries may be inferred.

The f in the equation 4.3 is proportional to the conjugacy class trace κ. One can actually read off the gauge field F from the boundary terms in the gauged WZW action of the brane as in [9] and obtain the relation between f and κ.

A possible objection to this procedure is that close to the singularity curvature effects could become large, and invalidate the use of the Born-Infeld action. The leading curvature corrections to the Born-Infeld action is discussed in [24] (we refer to the book and the references therein for further discussion). These take the form

$$S_{DBI} = \int e^{-\Phi}\sqrt{-det(\hat{g}+F)}(1 - \mathcal{R}_{abcd}\mathcal{R}^{abcd} + 2\hat{\mathcal{R}}_{ab}\hat{\mathcal{R}}^{ab})\,. \tag{4.4}$$

A calculation reveals that both for the Pf-theory and the black hole, the two curvature terms cancel. Thus at least to this order in α' the curvature effects do not play a role in this discussion.

In this section, we shall outline the various regular conjugacy classes of $\mathrm{SL}(2,\mathbb{R})$ and their projections to the coset which lead to the space-filling brane world-volumes. We shall work with the (u,v) co-ordinates throughout, which is suitable for describing the branes in the $1 - uv > 0$ region.

1. $\mathrm{Tr}(g) = 2$

   (a) $g_0 = \left(\begin{smallmatrix} 1 & 1 \\ 0 & 1 \end{smallmatrix}\right)$
   By explicitly conjugating g_0 by the element $h = \left(\begin{smallmatrix} a & u \\ -v & b \end{smallmatrix}\right) \in \mathrm{SL}(2,\mathbb{R})$, we get the following expression for points in the conjugacy class.

$$p = h\, g_0\, h^{-1} = \begin{pmatrix} a'v' + 1 & (a')^2 \\ -(v')^2 & 1 - a'v' \end{pmatrix} \tag{4.5}$$

   Thus, from the point of view of the coset, we have

$$\begin{aligned} u &= (a')^2 \\ v &= (v')^2 \end{aligned} \tag{4.6}$$

   Hence this "brane" fills out all of the $(u \geq 0, v \geq 0)$-region of the coset (because a' and v' range over all of the real line in $\mathrm{SL}(2,\mathbb{R})$).

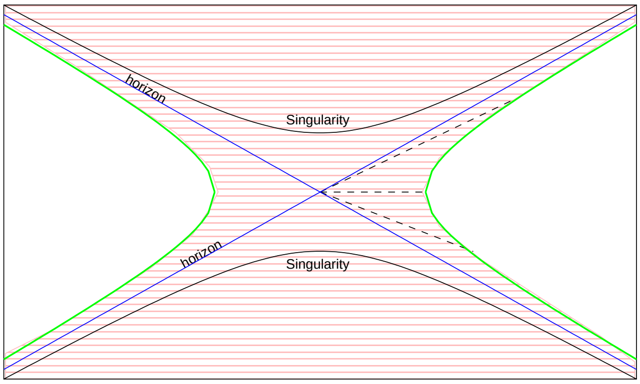

Figure 4: In-falling D-strings.

(b) $g_0 = \left(\begin{smallmatrix} 1 & 0 \\ 1 & 1 \end{smallmatrix}\right)$ In this case, the resultant brane covers the region $(u \leq 0, v \leq 0)$.

(c) $g_0 = \left(\begin{smallmatrix} 1 & -1 \\ 0 & 1 \end{smallmatrix}\right)$ In a similar manner, the points of the conjugacy class are

$$p = h\, g_0\, h^{-1} = \begin{pmatrix} -a'v' + 1 & -(a')^2 \\ (v')^2 & 1 + a'v' \end{pmatrix} \tag{4.7}$$

Thus, from the point of view of the coset, we have

$$\begin{aligned} u &= -(a')^2 \\ v &= -(v')^2 \end{aligned} \tag{4.8}$$

i.e. $u, v \leq 0$.

(d) $g_0 = \left(\begin{smallmatrix} 1 & 0 \\ -1 & 1 \end{smallmatrix}\right)$ In this last case we get the region $u, v \geq 0$.

We also have conjugacy classes corresponding to the negatives of the above g_0 which project to similar regions.

2. $|\,\mathrm{Tr}(g)| > 2$

In $\mathrm{SL}(2, \mathbb{R})$, this c.c. forms one connected hyperboloid.

Let $g = \left(\begin{smallmatrix} x+y & u \\ -v & x-y \end{smallmatrix}\right)$, and we are fixing the trace $2|x| > 2$. The determinant condition is

$$uv = (1 - x^2) + y^2 \tag{4.9}$$

Thus, as we vary y, this brane covers the region $uv \geq (1 - x^2)$. This brane extends into the physical region $uv < 0$, but also covers all of the region behind the singularity.

3. $|\,\mathrm{Tr}(g)| < 2$

In this case, from the above determinant condition, we see that $(1 - x^2) > 0$, which means that the brane covers the region $uv \geq (1 - x^2) > 0$ i.e. is entirely behind the horizon.

In the last two cases, we get the same world-volumes for either sign of the trace x.

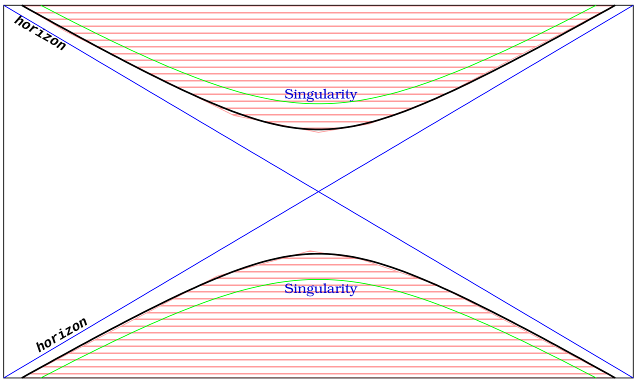

Figure 5: D2-branes behind the horizon.

Thus we have a family of D1-branes labelled by a single parameter κ corresponding to the trace of g_0 (or equivalently, f which is the world-volume electric field strength). The brane world-volumes are bounded by the singularity at one end on account of the F-field blowing up (it may be noted that F blows up by virtue of its dependence on the metric). At the other end they are bounded by the hyperbola $uv = 1 - \kappa^2$ in regions I-IV, and by $uv = 1 + f^2$ in regions V,VI.

The remarkable thing about these branes is that all the world-volumes have definite boundaries. A similar situation is seen to occur in the (euclidean) parafermion theory. In the asymptotically flat region I,II, since the world-volume gauge field remains finite, it is puzzling that the world-volume has a boundary.

The D1-branes which extend into region I of the black hole geometry represent world-volumes of D-strings which are emitted by the white hole and fall back into the black hole. From the point of view of an asymptotic observer, these D-strings stretch out from the horizon $\rho = 0$ in the far past, extend to a maximum length $\rho_{\max} = 1 - \kappa^2$ at $t = 0$ and then collapse back to the horizon (in the figure figure 4, the dashed lines represent the D-strings). Thus these are time dependent and physically observable to an asymptotic observer.

5. Comparison with the Pf-theory and euclidean black hole

As we remarked in the introduction, all the branes in the euclidean black hole geometry constructed in [1, 25] have their counterparts in the lorentzian background. However, the lorentzian case has several BCFT's that do not bear a euclidean continuation. One can perform an analysis of the gauged WZW model that leads to the euclidean hole (or the Pf-theory) along the lines of this work ([8]) and check this correspondence.

The $0 < uv < 1$ region can be analytically continued to the Parafermion theory, whose branes are discussed in [13]. As we discuss below, all the branes in that theory can be analytically continued subject to a few modifications.

One point to be noted is that in our work which is at the classical level, we have not investigated questions about quantization conditions on the branes. In both the euclidean theories, the branes are quantized — a precise comparison of the parameters labelling the branes will depend on a study of the isometry of the black hole geometry.

5.1 D-instantons

The allowed D(-1)-branes in the lorentzian black hole are at the horizon $u = v = 0$ and a one parameter family all located on the singularity $uv = 1$.

The $u = v = 0$ brane is the continuation of the single D0-brane in [1] which sits at the tip of the cigar. The ones on the singularity are not seen in the cigar geometry.

The branes on the singularity are the continuations of the A-branes of [13] which are all located on the boundary of the Pf-target space (which is also a curvature singularity). In that case, there is a discrete family because of quantization effects (the rotation isometry of the cigar is anomalous and a Z_k subgroup is preserved in the quantum theory).

The single brane at $u = v = 0$ corresponds to the single B-brane in the Pf-theory which is located at the centre of the disk.

5.2 D0-branes

The embedding equation of the D0-brane world volumes when analytically continued to the euclidean case maps into corresponding branes in the euclidean geometry [1]

$$\sinh r \cos\theta = 2\kappa$$

In the euclidean case, we have two parameter family of these branes. For reasons similar to the Pf-theory (i.e. the rotation isometry is anomalous) one of the labels is an integer. In the lorentzian case, we have two real parameters. This family of branes, in both cases, is in one-to-one correspondence with the primaries of these theories, in accordance with the Cardy-correspondence.

Similarly, the embedding equation can be continued to the Pf-theory also. In this case, we need to either use co-ordinates appropriate to the region $0 \leq uv \leq 1$ (in $\mathrm{SL}(2,\mathbb{R})$ charts). These D0-branes then map into the D1-branes of [13].

5.3 D1-branes

In the euclidean black hole case, we have a family of branes labelled by an integer, which cover the entire cigar as in [1] and also another set as described recently in [25] which cover a region near the tip of the cigar.

These branes are the analytic continuations of the D1-branes of section 4.3. The latter set corresponds to those c.c. with trace > 2. However, in the lorentzian theory, we do not have any D-strings whose world-volume covers the whole of the $uv < 0$ region. It is tempting to relate the former set of branes in the euclidean theory to those D-strings with trace ≤ 2 (If one uses the description of the *euclidean* black hole as a gauged WZW model, then these space-filling branes are indeed obtained by projection of those $\mathrm{SL}(2,\mathbb{R})$ c.c. with $\mathrm{Tr}(g) < 2$).

In the Pf-theory, there is a one (discrete) parameter family of B-branes, which are concentric discs. These are the analytic continuation of the D-strings. However, one difficulty is that the B-branes of [13] do not all reach the boundary of the disc while the ones in the black hole cover the entire $0 \leq uv \leq 1$ region under a naive analytic continuation. However, in both cases the branes terminate when the F-field on the world-volume blows up.

6. Duality relations between the various branes

The dual geometry in each (i.e. lorentzian and euclidean) is obtained by gauging the *vectorial* U(1) as opposed to the axial U(1). In global co-ordinates, as we have discussed briefly in section 3, this gauging implies that the geometry is now described by the (a, b) co-ordinates. Again $ab = 0$ is the horizon and $ab = 1$ is the singularity.

In the lorentzian case, if we restrict ourselves to the $\mathrm{SL}(2, \mathbb{R})$ co-ordinate chart relevant to the asymptotically flat region of the black hole, the dual geometry is the region behind the singularity of the original black hole.

Since the target space is the same in the dual description, one could ask how are the D-branes obtained in the vectorial gauging. For this purpose, it then suffices to examine the various conjugacy classes in $\mathrm{SL}(2, \mathbb{R})$ and see how they project to the (a, b) plane. The two projections, onto the (u, v) and (a, b) are then to be understood as being dual to each other. However, under duality the A-type and the B-type branes are exchanged i.e. the branes obtained from $\mathrm{Tr}(g) = k$ are dual to those from $\mathrm{Tr}(\omega g) = k$. This is because the automorphism ω takes the H-subgroup to its inverse (and the A-type branes have been defined to be those that are *not* invariant under the target space isometry).

This however is the same duality relations one obtains in CFT terms i.e. twined conjugacy classes are T-duals (B-branes) of the regular conjugacy classes (A-branes) (the brane labels are as appropriate for the vectorially gauged situation).

For instance, the D0-branes in regions I and II are obtained from the twined conjugacy class of $\mathrm{SL}(2, \mathbb{R})$. Under T-duality, these regions are mapped to V and VI and the twined c.c. become regular conjugacy classes. These latter are defined by $\mathrm{Tr}(g) = a + b = 2\kappa$ and are invariant under the (vectorial) symmetry being gauged. Note that for various values of the trace, the regular conjugacy classes have fairly non-trivial geometries in $\mathrm{SL}(2, \mathbb{R})$. But, their *projection* to the coset always yields straight lines in the (a, b) co-ordinates as is clear from the equation $a + b = 2\kappa$. and hence we again obtain D0-branes. And as before, we can translate these thus giving us a two parameter family.

There is a special case however; the identity conjugacy class sits at $a = b = 1$ giving point-like D-branes (upon including the translations we get a one parameter family) as we have already discussed.

In the (a, b) co-ordinates the twined conjugacy classes of $\mathrm{SL}(2, \mathbb{R})$ give rise to the D1-branes. In this case the determinant condition gives $ab + v(v + 2\kappa) = 1$, which implies $ab \leq 1 + \kappa^2$. Thus it covers a region behind the singularity from the point of view of the original geometry. In the original description, we reasoned that the D-strings in regions V,VI had to terminate at $uv = 1 + f^2$ because beyond $uv > 1 + f^2$ there was no solution to the B-I equations for F. It is very interesting that the blowing up of F is “T-dual” to a F finite situation (this also suggests that $f = \kappa$).

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The regions III and IV of the black hole geometry are "self-dual" (in the sense the Parafermion theory is self-dual)

7. Discussion

To summarise, we have performed a detailed analysis of the various boundary conditions that preserve some part of the current algebra symmetry of the lorentzian black hole CFT. We have found three kinds of D-branes in this background: D(-1) branes, D0-branes, and D-strings, several of these occurring in families. The D0's and D1 branes, from the point of view of an observer in the asymptotic (flat) region of the black hole appear to come out of the horizon to a maximum radial distance and then fall back to the horizon. In global co-ordinates these are emitted by the white hole and absorbed by the black hole.

We then performed a mini-superspace analysis of the world-volume theories, and found that the question of the spectrum is not resolved simply. This is because the allowed open string modes depends on what boundary conditions we impose at the singularity (for instance).

We then compared our results with the branes in the euclidean black hole and the Pf-theory, and showed that all the branes considered in these euclidean theories have their counterparts in the (appropriate regions) of the lorentzian black hole.

We conclude by enumerating a number of questions that naturally arise as a result of this study.

1. Boundary states: The most important question is to construct the wavefunctions that describe these branes. This can be achieved in many ways, and perhaps by a judicious combination of several of the following.

   We could follow the procedure adopted in [13] together with the known one-point functions of branes in $SL(2,\mathbb{R})$ to derive the boundary states. This procedure is fraught with some difficulty because of the non-compactness of the $SL(2,\mathbb{R})$ CFT.

   Another method is to use the "shape of branes" argument [12]. The one-point functions must be such that when projected onto closed string states which have δ-function wavefunctions, the amplitudes must be supported on the world-volumes of the branes. Since we know the geometry, we can derive the one-point functions in the large k-limit, when geometry is reliable by using the expansion for the delta function in terms of the closed string vertex operators.

2. The curvature singularity: It will be of great interest to study these BCFT's or equivalently the boundary states to understand the singularity. The D-branes we have discussed seem to exist on both sides of the singularity; in particular the D0-branes do not seem to "see" the singularity at all. It remains to be seen how this semi-classical result gets modified in a more careful quantum analysis of the geometry.

3. What is the nature, if any, of the closed string radiation produced by these branes? The D-branes we have constructed are explicitly time-dependent. Once the boundary

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states are known, we can study the nature of the radiation emitted (and absorbed) by these, and compare with the Liouville case. The presence of the black (and white) hole makes such a calculation interesting. Note that while in regions I and II the metric is time independent, the metric behind the horizon (regions III and IV) is time *dependent*. It must be noted that the nature of the open string spectrum (and therefore the stability) of these branes is somewhat delicate since these are time-dependent.

4. D0-branes: The D0-branes are described by embedding equations $\sinh\rho\cosh t = k$ which resemble that of the lump solutions of [26]. It will be of some interest to investigate this similarity further, especially the homologue of tachyon matter in the black hole geometry. This question could be pursued perhaps along the lines of [27] where the tachyon matter state has been related to an array of branes in imaginary time.

5. Another natural and important question of study would be the extension of these results to the supersymmetric versions of these coset theories. The black hole constructions of [28] and [29] has this CFT as a building block. In these cases, one can hope that these branes are useful to study the black hole singularity in a four dimensional context.

6. The relationship to the c=1 phase space? The relationship between $c = 1$ string theory and 2D black hole is spelt out in Das [30] and Dhar et al. [31]. In particular, the former work argues that the macroscopic loop equations of the Matrix model is a transform of the Wheeler-DeWitt equation of the black hole geometry. This is one way of relating these branes to that of the $c = 1$ string theory.

   Another interesting question to pursue is the nature, if any, of w_∞ charges that are carried by these branes in analogy with [32].

7. Recent works [33, 34, 35] have discussed D-branes in the NS5-brane background with space-time behaviour which is similar to our D-branes.

8. Several unusual features of these D-branes such as the bounded world-volumes, and the fact that they are "bound" to the black hole deserve to be studied further.

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A. Co-ordinate systems for $\mathrm{SL}(2,\mathbb{R})$

Every matrix $g \in \mathrm{SL}(2,\mathbb{R})$ with all entries nonzero can be written as a product [36]

$$g = d_1(-e)^{\epsilon_1} s^{\epsilon_2} p\, d_2 \tag{A.1}$$

where $d_{1,2} = \mathrm{diag}(e^{\theta_{1,2}}, e^{-\theta_{1,2}})$ and $\theta_{1,2} \in (-\infty, \infty)$, e is the identity matrix, s is the matrix $\left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right)$, p is one of the two matrices

$$p_1 = \begin{pmatrix} \cosh\rho & -\sinh\rho \\ -\sinh\rho & \cosh\rho \end{pmatrix} \qquad \rho \in [-\infty, \infty)$$

or

$$p_2 = \begin{pmatrix} \cos\rho & \sin\rho \\ -\sin\rho & \cos\rho \end{pmatrix} \qquad \rho \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$$

and $\epsilon_{1,2} = 0, 1$.

In a similar manner, the matrices in $\mathrm{SL}(2,\mathbb{R})$ with at least one zero entry can be written as a product

$$g = d\,(-e)^{\epsilon_1}\, s^{\epsilon_2} \begin{pmatrix} 1 & 0 \\ x & 1 \end{pmatrix} s^{\epsilon_3} \tag{A.2}$$

where $d = \mathrm{diag}(e^{\phi}, e^{-\phi})$ and e and s are as above.

The gauge symmetry that leads to the lorentzian black hole acts as $\theta_{1,2} \to \theta_{1,2} + \epsilon$, and the time co-ordinate t of the black hole geometry is related to the θ as $t = (\theta_1 - \theta_2)$. It is then easy to see how the various co-ordinate charts project down (upon gauging) to cover different regions of the black hole coset.

For instance, setting $p = p_1\, \epsilon_{1,2} = 0$ gives us $\mathrm{SL}(2,\mathbb{R})$ matrices of the form

$$g = d_1 \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} d_2$$

Gauging sets $d_1 = d_2$ and projects to the (u, v) co-ordinates. The matrices above are then seen to cover the $uv < 0$ regions of the black hole geometry. Note that $p = p_1$, $\epsilon_1 = 1$, $\epsilon_2 = 0$ also covers the same region of the coset. Similarly, in every co-ordinate chart, multiplication by the identity matrix yields another copy of the same region in the coset (upon gauging). Thus the gauged sigma model gives two copies of the black hole geometry [18].

Thus we obtain the following covering diagram figure 6 (in this diagram we will omit ϵ_1 since as discussed above, we simply get another copy of the coset if we include $-I$ factors). The matrices in $\mathrm{SL}(2,\mathbb{R})$ with zero entries cover the horizon lines $uv = 0$ of the coset and are not indicated in the figure (figure 6). The singularity is the dark (black) line in the figure, and the region between the horizon and the singularity is covered by the two charts with p_2 type of matrices.

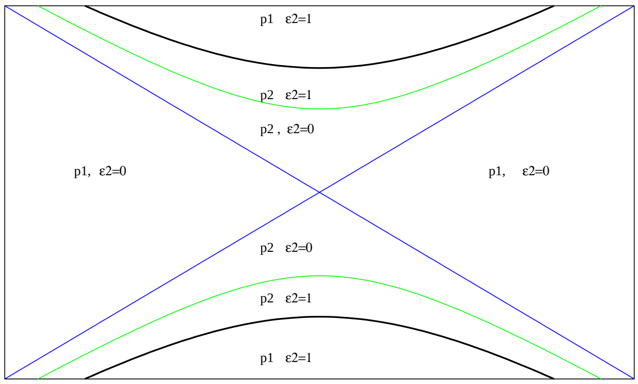

Figure 6: The black hole in $\mathrm{SL}(2,\mathbb{R})$ charts.

B. The BCFT action of the branes

In this section, we shall briefly discuss the action governing these boundary conformal field theories. For details, we refer to [5, 6, 9, 8]. Here, we shall consider the vectorial gauging of a subgroup $H \subset G$ (we can freely switch between the axial/vector gauging because both lead to the same target space).

The action for a gauged WZW model on a worldsheet without a boundary is

$$\begin{aligned} S &= \frac{k_G}{4\pi}\left[\int_\Sigma d^2z\, L^{kin} + \int_B \omega^{WZ} + S^{\text{gauge}}\right] \\ &= \frac{k_G}{4\pi}\left[\int_\Sigma d^2z\, \mathrm{Tr}(\partial g\bar\partial g^{-1}) + \int_B \frac{1}{3}\mathrm{Tr}(g^{-1}dg)^3\right] + \\ &\quad + \frac{k_G}{2\pi}\int_\Sigma d^2z\ \mathrm{Tr}\left(\bar A\partial g g^{-1} - A g^{-1}\bar\partial g + \bar A g A g^{-1} - A\bar A\right). \end{aligned} \tag{B.1}$$

The Wess-Zumino form ω^{WZ} is integrated over a 3-manifold B whose boundary $\partial b = \Sigma$, the closed-string worldsheet.

In the case of a boundary conformal field theory, since Σ itself has a boundary (in our case Σ is a disk) there is no B whose boundary is $\partial B = \Sigma$. This action is then modified by gluing an auxillary disk D to the world-sheet to get a surface without a boundary, and modifying the action such that the contributions from the disk D cancel. A further requirement that the various embeddings the disk in G should give the same contribution then restricts the allowed boundary values further. This is achieved as follows.

For the A-type branes, the worldsheet field $g(z,\bar z)$ is extended to the disk D by requiring that on D and on the boundary $\partial\Sigma$ g is restricted to the set $g \in C_f^G\, C_l^H$, i.e. a product of conjugacy classes $C_f^G = kfk^{-1}$ of G and $C_l^H = plp^{-1}$ of H respectively.

The additional term has the form

$$-\frac{k_G}{4\pi}\int_D \Omega^{(f,l)}(k,p) = -\frac{k_G}{4\pi}\int_D \left[\omega^f(k) + \mathrm{Tr}(dc_2c_2^{-1}c_1^{-1}dc_1) + \omega^l(p)\right]. \tag{B.2}$$

Here $c_1 = kfk^{-1}, f,k \in G$ and $c_2 = plp^{-1}, p,l \in H$ and $\omega^g(h)$ is the Recknagel-Schomerus two form

$$\omega^g(h) = \mathrm{Tr}(h^{-1}dhgh^{-1}dhg^{-1})\,.$$

The two form $\Omega^{(f,l)}(k,p)$ in the above integral has the property that $d\Omega^{(f,l)}(k,p) = w^{WZ}(g)$ when g is restricted to the set $C_f^G\, C_l^H$; hence on the auxillary disk the two terms cancel. The boundary conditions on g are that it is restricted to a product of the conjugacy classes, except that the gauging identifies field configurations $g \sim h\, g\, h^{-1}$ for $h \in H$. Thus, the worldsheet boundaries are restricted to the set $C_f^G C_l^H$ projected to the coset G/H.

The case of the B-type branes is similar [9, 8]. In this case the boundary conditions on the field g restricts the endpoint to lie in product of *twined* conjugacy classes $C_f^{\alpha,G}\, C_l^{\alpha,H}$ where α is an automorphism of G. Correspondingly, the additional terms B.2 that were required to cancel the contribution of the WZ three form on the auxillary disk have a different form. In this case the extra pieces of the action are

$$-\frac{k_G}{4\pi}\int_D \Omega^{(f,l)}(k,p) = -\frac{k_G}{4\pi}\int_D \left[\omega^f(k) + \mathrm{Tr}(dc_2 c_2^{-1} c_1^{-1} dc_1) + \omega^l(p)\right] \qquad \text{(B.3)}$$

In this case, the two form ω is given by

$$\omega^g(h) = \mathrm{Tr}(\alpha(h^{-1}dh)gh^{-1}dhg^{-1})$$

and $c_1 = \alpha(k)fk^{-1}, f, k \in G$ and $c_2 = \alpha(p)lp^{-1}, p, l \in H$ As before, on the set $C_f^{\omega,G}\, C_l^{\omega,H}$, $d\Omega^{(f,l)}(k,p) = w^{WZ}(g)$ and the two terms cancel.

In a similar manner, we can work out the case when the gauged symmetry is the axial embedding of H. As one could perhaps guess, the A- and B-type branes are interchanged. In our case, $G = \mathrm{SL}(2,\mathbb{R})$ and $H = \mathrm{U}(1)$, and both the axial and vectorial gauging give rise to the same target space geometry.

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